Bharatiya Vidya Bhavan's

SARDAR PATEL COLLEGE OF ENGINEERING

(Government Aided Autonomous Institute)

Munshi Nagar, Andheri (W) Mumbai – 400058

END SEM / ~~RE-EXAM~~ EXAMINATION MAY / ~~JUNE~~ 2025

Program: First Year M. Tech Structures Sem II

Duration: 3 Hrs.

Course Code: PC-MTSE201

Maximum Points: 100

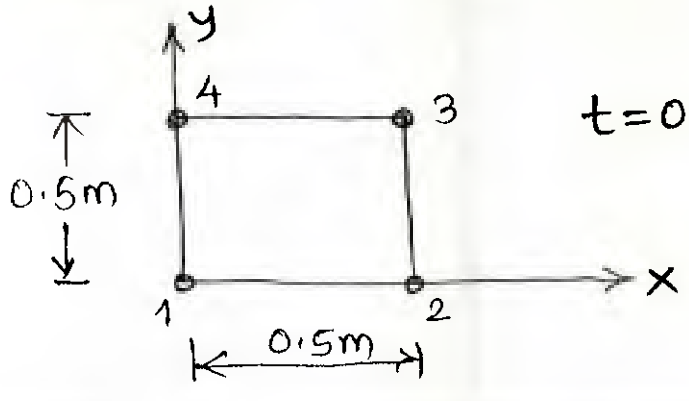
Course Name: Finite Element Analysis

Semester: II

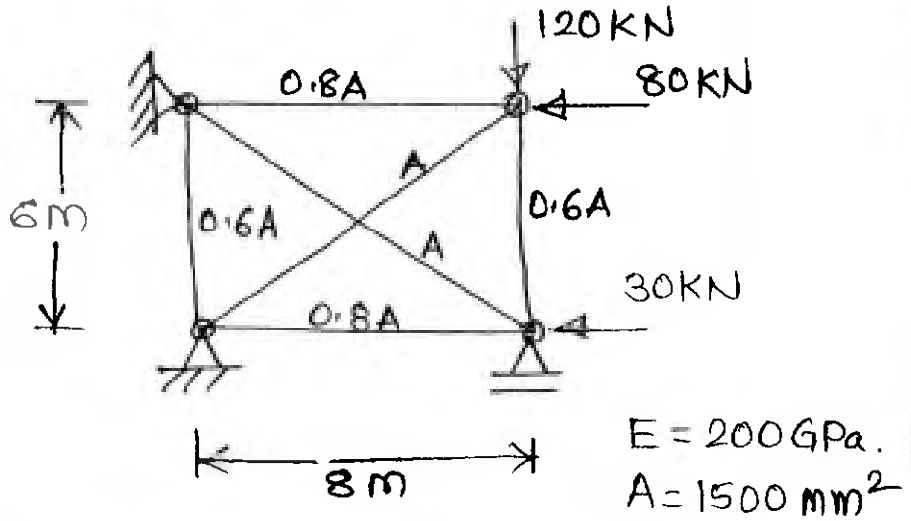
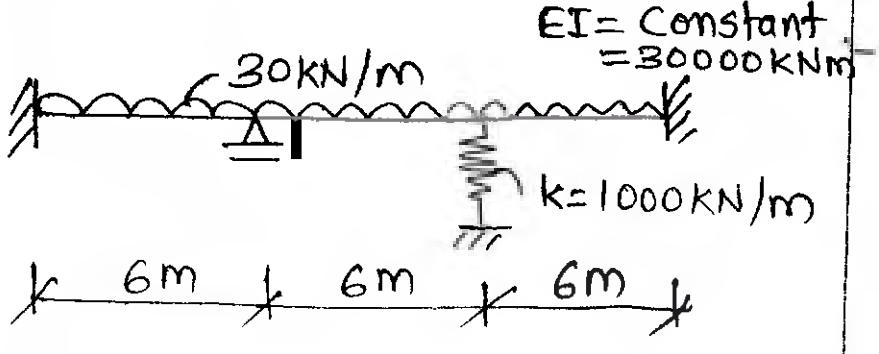
20/5/25

Instructions for the Students:

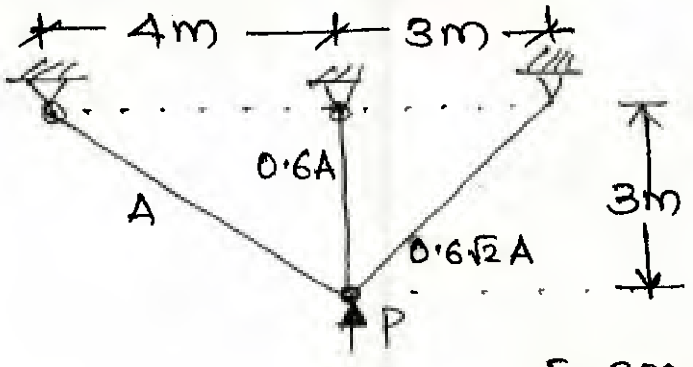
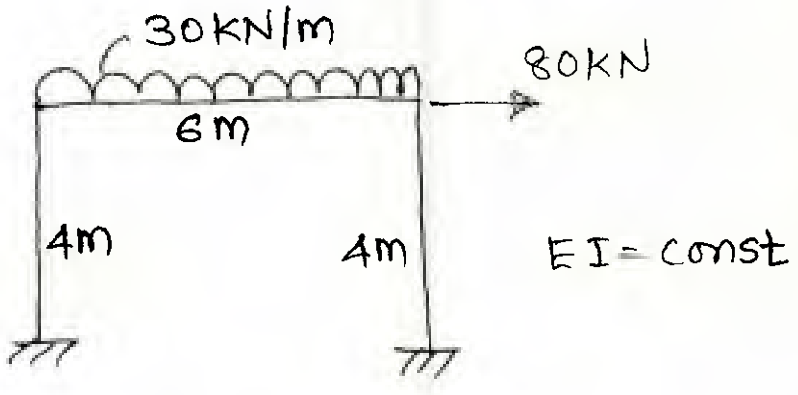
1. **Attempt any 05 (FIVE) questions**
2. **Assume suitable data if necessary and state it clearly.**
3. **Illustrate your answers with neat sketches wherever required.**

Q. No.	Questions	Points	CO	BL	Module No.
1 a	Write short note discretization of domain in finite element analysis.	10	2	2	2
1 b	Determine the interpolation functions for 8-noded Lagrange element in natural coordinate system.	10	2	4	3
2 a	Derive a stiffness matrix for a CST element having joint coordinates as follows: Node 1 (0,0), Node 2 (0.2m, 0), Node 3 (0, 0.2m). Assume the element as plane stress element with thickness of 0.020 m, E = 200 GPa and $\nu = 0.2$.	10	2	4	3,5
2 b	For the simply supported beam subjected to uniformly distributed load throughout, determine the maximum displacement using variational approach.	10	1	4	1
3 a	Determine the nodal load vector for a four noded rectangular element as shown in figure below. Note that the element is subjected to a concentrated load of 80 kN in X direction acting at a point (0.2m, 0.2m) and 60 kN in Y direction acting at centre of the element. 	10	2	4	3,5

END SEM / ~~RE-EXAM~~ EXAMINATION MAY / ~~JUNE~~ 2025

3 b	For the element shown in Q3a, if the joint displacements for the nodes 3, 4 are 0.003, 0.001 in x direction and 0.004, 0.001 in y direction respectively, determine element stresses and element strains at a point (0.2m, 0.2m). Take E = 200 GPa and ν = 0.2 and assume element in plane strain. Note that displacement at joint 1 & 2 is zero.	10	2	4	3,6
4	For the truss shown in figure below, using finite element techniques, determine member stiffness matrix, structure stiffness matrix, effective structure stiffness matrix, loading vector, displacement vector, element forces, element stresses and strains. 	20	3	4	3,4
5	For the continuous beam shown in figure below, using finite element techniques, determine element stiffness matrix, structure stiffness matrix, effective structure stiffness matrix, loading vector, displacement vector, member forces. 	20	3	4	3,4

END SEM / ~~RE-EXAM~~ EXAMINATION MAY / ~~JUNE~~ 2025

6	For the truss shown in below, using finite element techniques, determine member stiffness matrix, structure stiffness matrix, geometric stiffness matrix and buckling load for the truss.  E = 200 GPa ρ = 7850 kg/m³ A = 1000 mm²	20	4	4	3,7
7	For the plane frame shown in figure below, using finite element techniques, determine element stiffness matrix, structure stiffness matrix, effective structure stiffness matrix, loading vector, displacement vector, member forces.  	20	3	4	3,4

SARDAR PATEL COLLEGE OF ENGINEERING

(Government Aided Autonomous Institute)
Munshi Nagar, Andheri (W) Mumbai – 400058

~~END SEM~~ / RE-EXAM EXAMINATION ~~MAY~~ / JUNE 2025

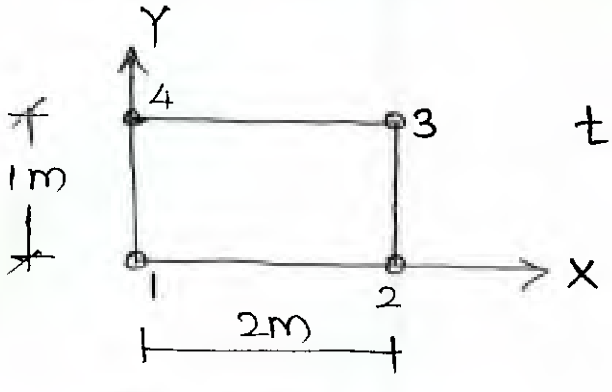
Program: First Year M. Tech Structures Sem I
Course Code: PC-MTSE201
Course Name: Finite Element Analysis

Duration: 3 Hrs.
Maximum Points: 100
Semester: II

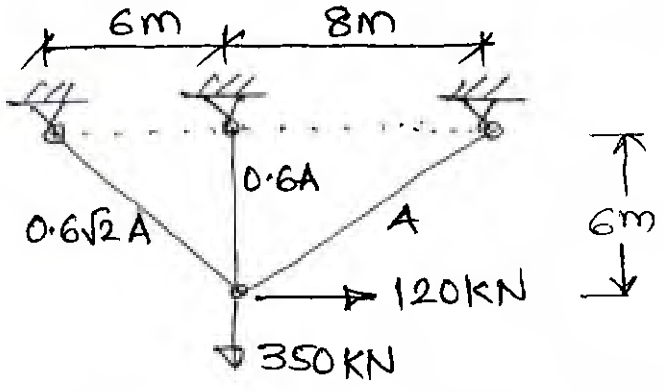
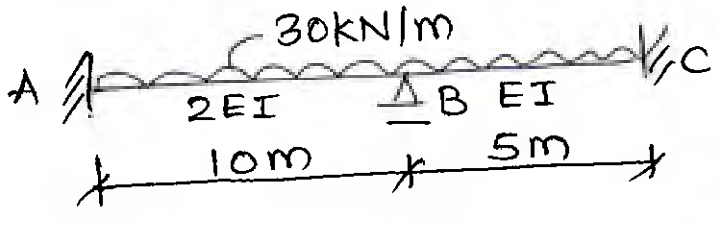
25/6/25

Instructions for the Students:

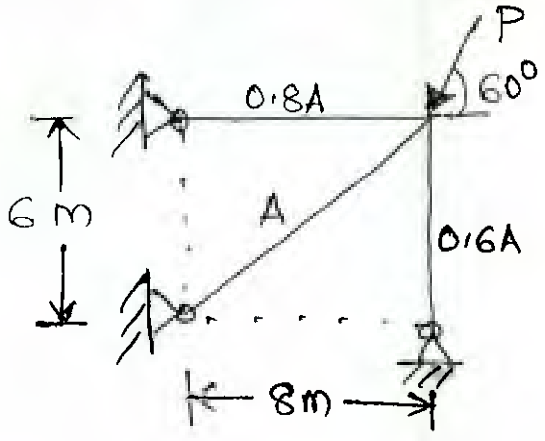
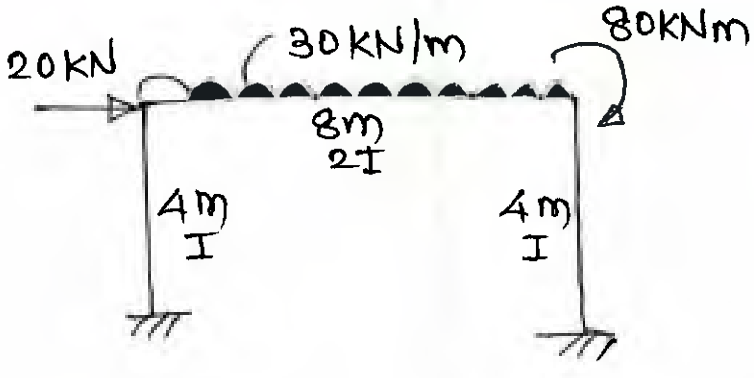
1. **Attempt any 05 (FIVE) questions**
2. **Assume suitable data if necessary and state it clearly.**
3. **Illustrate your answers with neat sketches wherever required.**

Q. No.	Questions	Points	CO	BL	Module No.
1 a	Write short note discretization of domain in finite element analysis.	10	2	2	2
1 b	Determine the interpolation functions for 6-noded Lagrange element in natural coordinate system.	10	2	4	3
2 a	Derive a stiffness matrix for a CST element having joint coordinates as follows: Node 1 (0,0), Node 2 (0.5m, 0), Node 3 (0.5m, 0.5m). Assume the element as plane strain element with thickness of 0.020 m, E = 200 GPa and ν = 0.25.	10	2	4	3,5
2 b	For the simply supported beam subjected to uniformly distributed load throughout, determine the maximum displacement using finite difference technique.	10	1	4	1
3 a	Determine the nodal load vector for a four noded rectangular element as shown in figure below. Note that the element is subjected to a concentrated load of 30 kN in X direction acting at a point (0.3m, 0.5m) and 40 kN in Y direction acting at a point (0.5m, 0.5m). 	10	2	4	3,5

~~END SEM~~ / RE-EXAM EXAMINATION ~~MAY~~ / JUNE 2025

3 b	For the element shown in Q3a, if the joint displacements for the nodes 1, 2, 3, 4 are 0.001, 0.002, 0.003, 0.002 in x direction and 0.002, 0.002, 0.001, 0.003in y direction respectively, determine element stresses and element strains at a point (0.5m, 0.7m). Take E = 200 GPa and ν = 0.25 and assume element in plane stress.	10	2	4	3,6
4	For the truss shown in figure below, using finite element techniques, determine member stiffness matrix, structure stiffness matrix, effective structure stiffness matrix, loading vector, displacement vector, element forces, element stresses and strains.  E = 200GPa A = 1250 mm²	20	3	4	3,4
5	For the continuous beam shown in figure below, using finite element techniques, determine element stiffness matrix, structure stiffness matrix, effective structure stiffness matrix, loading vector, displacement vector, member forces.  EI = 25000 KN-m² Note that Support B Settles down by 20mm.	20	3	4	3,4

~~END SEM~~ / RE-EXAM EXAMINATION ~~MAY~~ / JUNE 2025

6	For the truss shown in below, using finite element techniques, determine member stiffness matrix, structure stiffness matrix, geometric stiffness matrix and buckling load for the truss. 	20	4	4	3,7
7	For the plane frame shown in figure below, using finite element techniques, determine element stiffness matrix, structure stiffness matrix, effective structure stiffness matrix, loading vector, displacement vector, member forces. 	20	3	4	3,4

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Sardar Patel College of Engineering

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Munshi Nagar, Andheri (West), Mumbai – 400058.

End Semester ~~Re-Examination~~ ~~June 2024~~ May 2025

Max. Marks: 100 Civil w/m Sem. Exp. Sem II Duration: 3 Hours

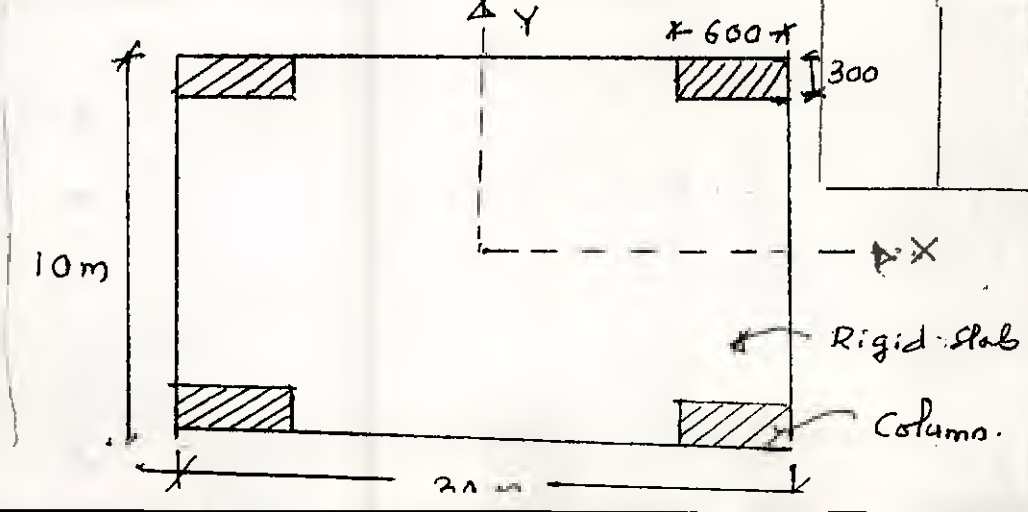
Class: M.Tech. Semester: II Program: Civil Engineering

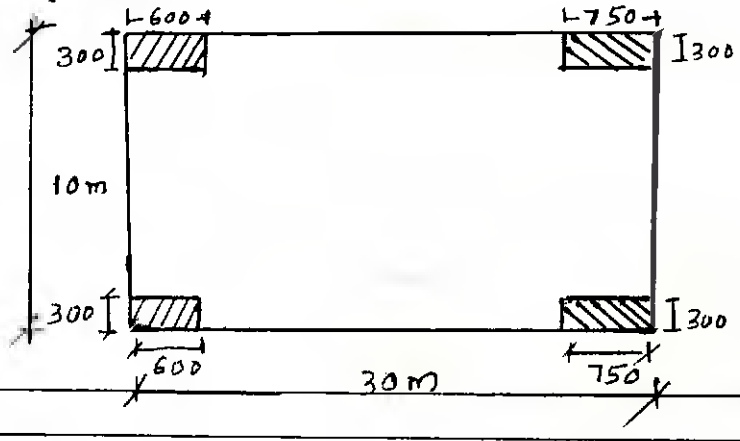
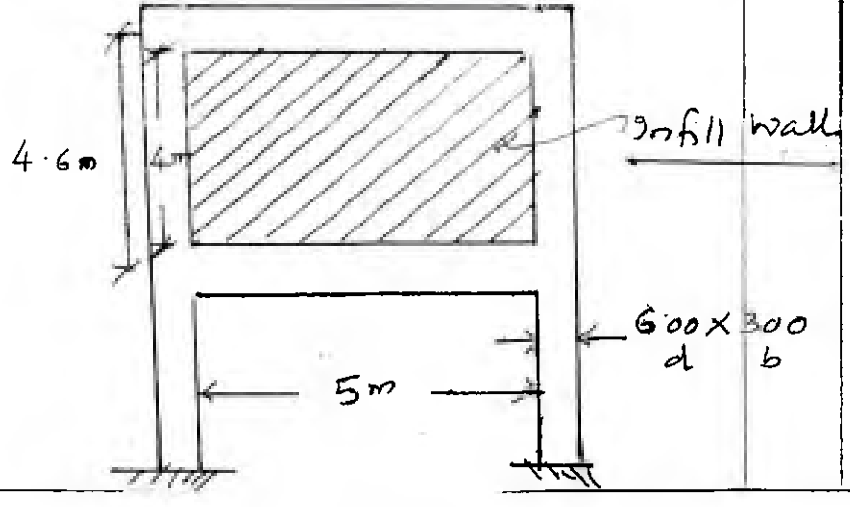
Name of the Course: Earthquake Engineering Course Code : PC-MST 202

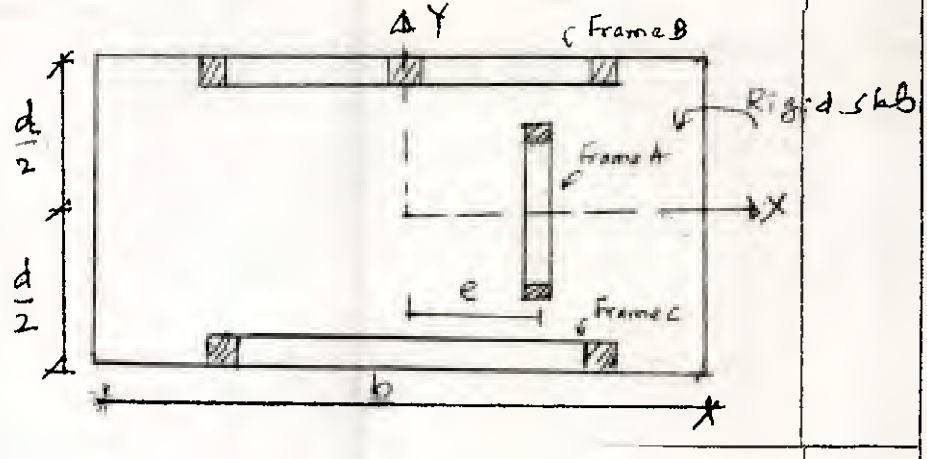
Instructions:

- Attempt any FIVE questions out of SEVEN questions.
- Answers to all sub questions should be grouped together.
- Figures to the right indicate full marks.
- Assume suitable data if necessary and state the same clearly

Question No		Max. Marks	Course outcome	Module No.
Q1 (a)	Answer the followings: (ii) What is ductility? Explain briefly the significance of ductility in earthquake resistant structures	3	4	6
	(iii) Briefly explain the Plate Tectonic Theory of an earthquake	4	1	2
	(iii) Explain briefly the structure of earth.	3	1	2
Q1 (b)	**(i)** A single-story structure with a rigid slab is supported on four corner columns as shown in the figure. The height of the structure is 6.0 m. In general, what will be the degrees of freedom for this structure? And specify these DoF. Calculate the structure's natural frequency for excitation in X and Y directions separately.	5	2	1
	(ii) If the system is subjected to harmonic ground acceleration with amplitude of 0.3g and excitation frequency of 20 rad/sec in X direction, evaluate the maximum lateral displacement of the slab. The weight on the slab is 200kg/m^2, uniformly distributed. Assume $\xi = 5\%$ and E= 2x10^4 N/mm^2.	5	2	1,4

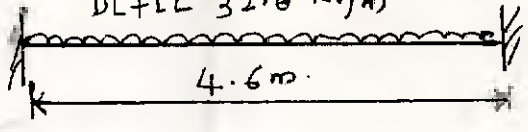


Q2 (a)	A rigid slab is supported on the four corner columns as shown in figure. Calculate the natural frequency and mode shapes for the structure. 	**12**	**2**	**1**
Q2(b)	If all the column sizes are same, calculate all the three natural frequencies	**4**	**2**	**1**
Q2(c)	If the above structure is subjected to an El Centro earthquake ground motion ground motion whose response spectra is shown in Figure 1 (i) only in X direction and (ii) Only in Y direction separately. Calculate the maximum displacements of slab for both cases. Assume damping ratio $\xi = 5\%$.	**4**	**3**	**4**
Q3 (a)	A two storey building frame with brick infill wall is shown in figure. Assuming the beams are rigid, calculate the lateral stiffness of the 2nd storey considering the effect of infill wall considering the infill was stiffness also as per IS 1893-2016. Compare the stiffness of the 2nd storey of frame with and without infill wall. 	**10**	**2,4**	**1.6**

	For URM infill walls without any opening, width w_{ds} of equivalent diagonal strut (*see* Fig. 7) shall be taken as: $$w_{ds} = 0.175\alpha_h^{-0.4} L_{ds}$$ where $$\alpha_h = h\left(\sqrt[4]{\frac{E_m t \sin 2\theta}{4E_f I_c h}}\right)$$			
Q3 (b)	The plan of one story building is as shown in figure. The structure consists of a roof idealized as a rigid diaphragm, supported on three frames A, B, and C as shown. The roof weight is uniformly distributed and has magnitude 200 Kg/m2. The lateral stiffness are Ky = 30000 KN/m for frame A and Kx =25000 KN/m for frame B and frame C Kx=30,000KN/m. The plan dimensions are b= 30 m d=20m. The height of building is 8m. e = 5 m (i) Derive the stiffness matrix (ii) If the structure is subjected to ground motion only in Y direction. write down the equations of motion for the system 	6 2	2 1	1 1
Q3(c)	As a special case, if Kx = 30,000 KN/m for both the frames B & C, and if the system is subjected to the ground motion only in Y direction, the response spectrum of which is shown in figure1. Determine the design value of lateral deformation, base shear and bending moment for the system	4	3	4

<table>
<tr><td>Q4(a)</td><td>What is the response spectrum? Explain the procedure to construct an elastic response spectrum for a given peak ground motion parameters including amplification factors</td><td>5</td><td>3</td><td>3</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Q4(b)</td><td>Explain the characteristics of ground motions</td><td>3</td><td>3</td><td>3</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Q4 (c)</td><td>A four-story school building special moment resisting frame has a story height of 4 m for the ground story and 3 m for the 2nd to 4th story. The building is located in Mumbai. The weight on the 1st to 3rd floor is 400 KN and on the 4th floor, it is 300KN. Using the equivalent static method, calculate the distribution of lateral loads and story shear. Assume soil strata as a medium. Use the response spectra given in Figure 2.</td><td>13</td><td>4</td><td>5</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Q5 (a)</td><td>Explain the three requirements of displacement design of structure for earthquake load as per IS 1893-2016.</td><td>4</td><td>4</td><td>5</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Q5 (b)</td><td>As per IS 1893-2016, how many mode need to be considered in the earthquake force calculation by Response Spectrum Method</td><td>2</td><td>4</td><td>5</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Q5 (c)</td><td>State the limitation of Equivalent static Method. As per IS 1893-2016, under what conditions the Equivalent static Method is permitted to use to calculate the earthquake forces.</td><td>2</td><td>4</td><td>5</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Q5 (d)</td><td>Using response spectrum method, calculate the seismic force on each floor of the frame whose pre vibration properties are given below. Use the following additional data: Z=0.24, I =1.5, R=5.0 and ξ = 5%. Assume foundation strata as soft soil and use response spectrum given in figure 3.

Assume the story height as 4m for all storey.
<table>
<tr><td>Story No.</td><td>Mass No.</td><td>Mass (t)</td><td>ω rad/sec</td><td colspan="3">Mode shapes</td></tr>
<tr><td></td><td></td><td></td><td></td><td>Φ_{i1}</td><td>Φ_{i2}</td><td>Φ_{i3}</td></tr>
<tr><td>1</td><td>1</td><td>50</td><td>14.52</td><td>1.00</td><td>2.160</td><td>3.313</td></tr>
<tr><td>2</td><td>2</td><td>50</td><td>31.05</td><td>1.00</td><td>0.893</td><td>-1.473</td></tr>
<tr><td>3</td><td>3</td><td>40</td><td>46.10</td><td>1.00</td><td>-1.042</td><td>0.410</td></tr>
</table></td><td>12</td><td>4</td><td>5</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Q 6(a)</td><td>Briefly explain the Response Spectrum characteristics</td><td>4</td><td>3</td><td>3</td></tr>
</table>

Q 6(b)	Briefly explain the following: (i) Explain briefly that as per IS 1893-2016 provisions, under what condition and which structures the vertical ground motion to be considered? ii) Explain the three requirements of displacement design of structure for earthquake load as per IS 1893-2016.	6	4	5
Q 6(c)	(i) A building having a non-uniform distribution of mass is shown in figure 4. Locate its center of mass (ii) The plan of a simple one-storied building shown in figure. The stiffness of each column is shown in figure5. Obtain the center of stiffness. (Centre of Rigidity).	4	4	5
Q 6(d)	The first floor plan of a building with shear walls is as shown in figure 6. The plinth level plan is also same. Calculate the fundamental period of the building as per the provision of 7.6.2, of IS 1893-2016 both in X and Y direction. The total height of building is 24 m.	4	4	5
Q 6(e)	For the SMRFs idealized as shear building with rigid girders shown in figure7, investigate whether the building structure has soft story. The height of first story is 4.5 m and that of remaining is 3.0 m.	2	4	5
Q 7(a)	A beam of a frame is shown in figure 8 with limiting moment of resistance at both ends and for both sway from left right and right to left. Calculate the design shear force as per IS 13920 -2016. The calculated factored shear force is 142 KN  For sway from Left to right $M_{u,lim}^{AS} = 140$ KNm $M_{u,lim}^{BH} = 120$ KNm For sway from right to left $M_{u,lim}^{AH} = 200$ KNm. $M_{u,lim}^{BS} = 80$ KNm	5	4	6
Q 7(b)	For a column of cross section 530mm x 300mm calculate the special confinement reinforcement (i.e diameter and spacing of ties as a special confinement reinforcement) as per IS	5	4	6

	13920-2026. The grade of concrete is M 25 and that of steel is Fe 415, consider clear cover to the ties is 60 mm. Also calculate the length l_0 of special confinement reinforcement. The clear height of column is 4.0 m.			
Q 7(c)	Explain the provisions of IS 13920 for (i)Beams: General provisions, longitudinal reinforcement and web reinforcement.	**5**	**4**	**5,6**
Q 7(d)	Explain in brief the different types of bracings with neat sketches that can be used as lateral load resisting systems in steel structures	**5**	**4**	**7**

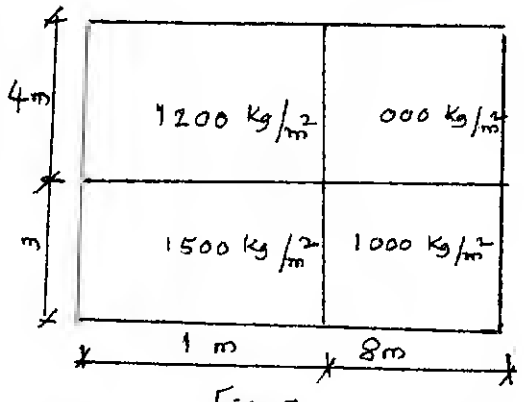


Fig. 4

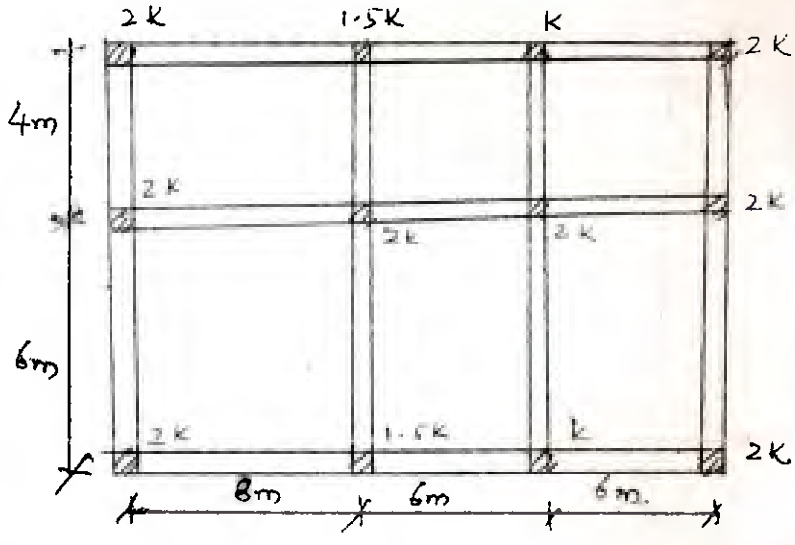


Fig. 5

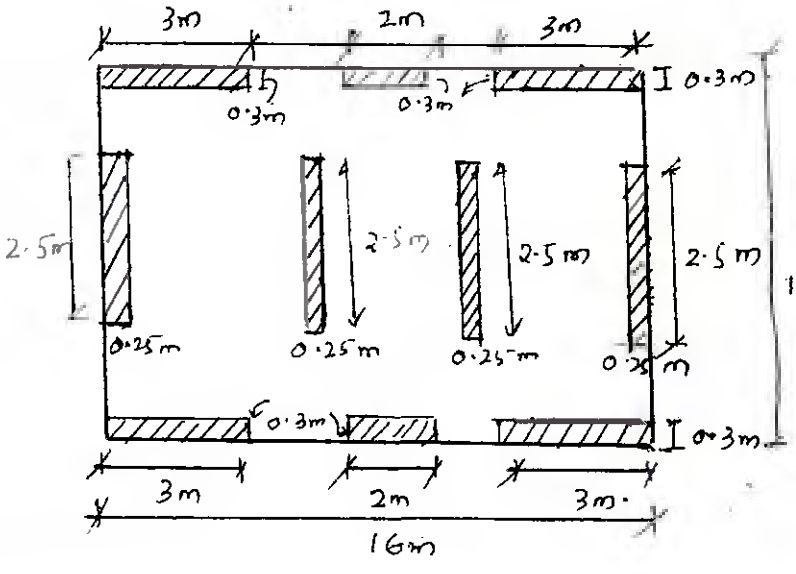


Fig. 6

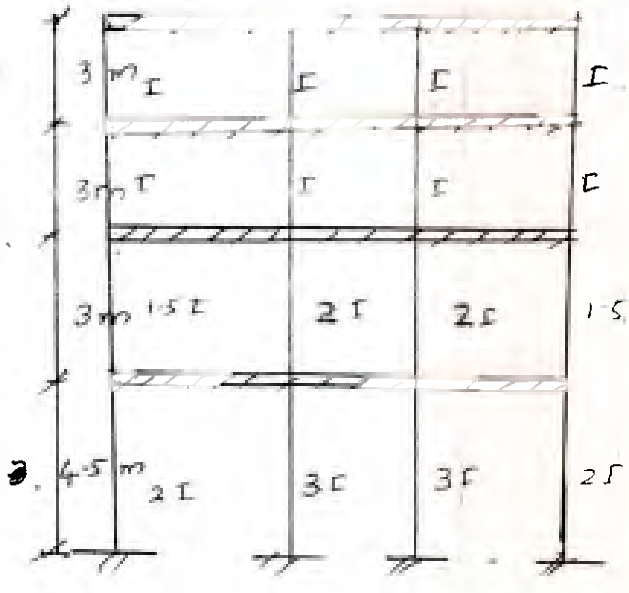
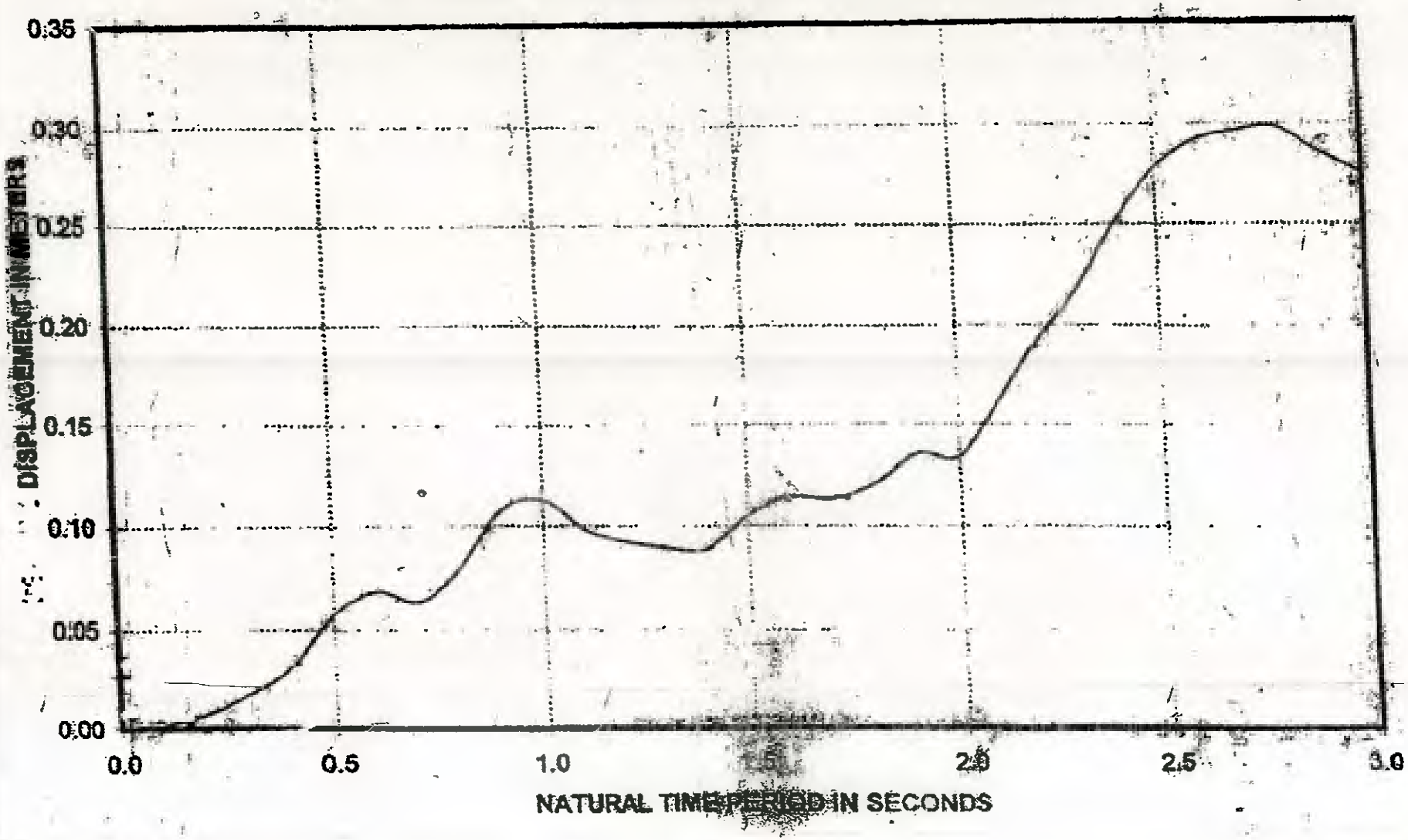


Fig. 7

DISPLACEMENT RESPONSE SPECTRA
FOR EL-CENTRO EARTHQUAKE FOR 5% DAMPING PGA = 0.32g



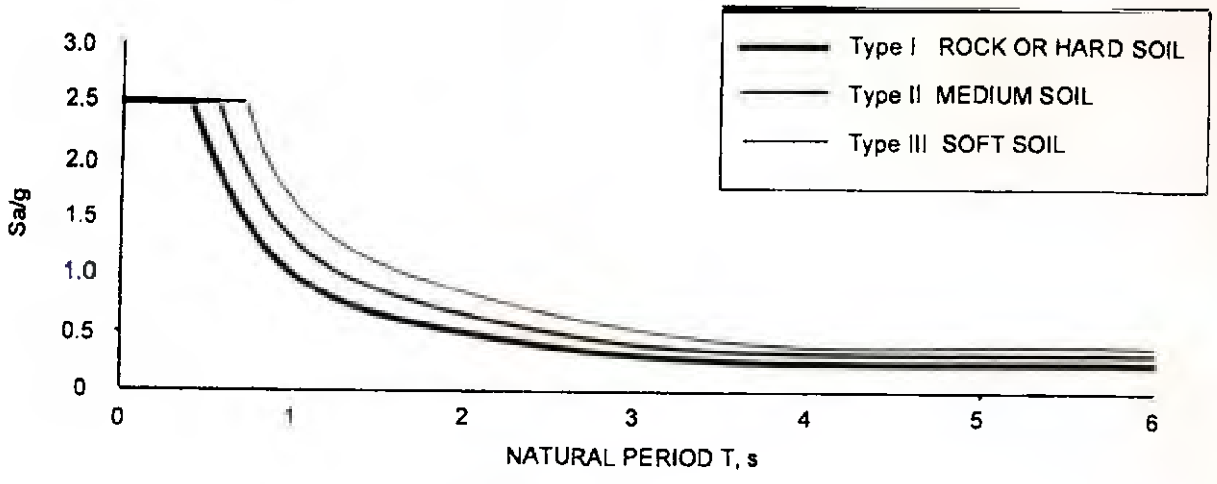


Fig. 2 2A SPECTRA FOR EQUIVALENT STATIC METHOD

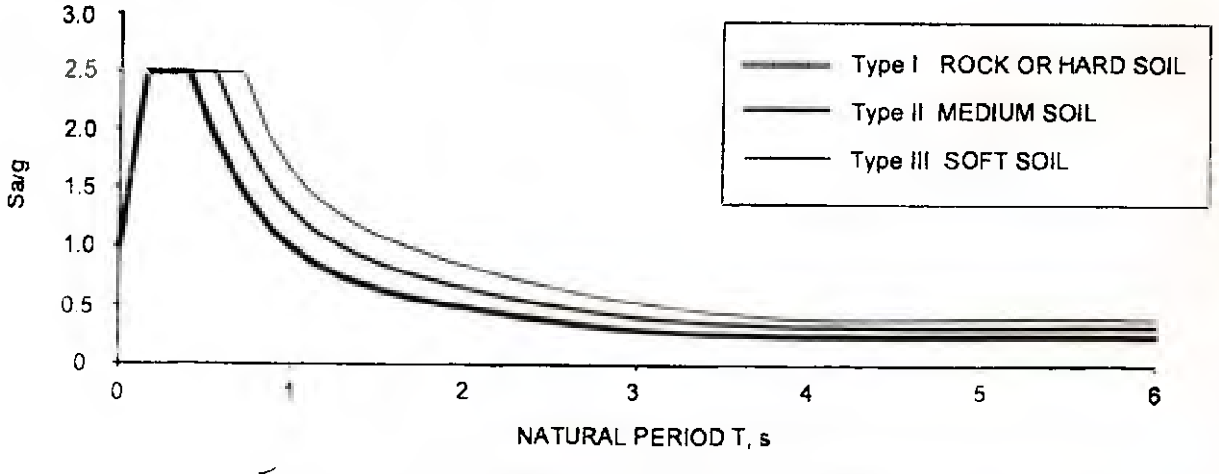


Fig 3 2B SPECTRA FOR RESPONSE SPECTRUM METHOD

FIG. 2 DESIGN ACCELERATION COEFFICIENT (S_a/g) (CORRESPONDING TO 5 PERCENT DAMPING)

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Munshi Nagar, Andheri (West), Mumbai – 400058.

Re – ~~End-Semester~~ Examination June - 2025

~~May - 2025~~

Max. Marks: 100 Duration: 3 Hours

Class: M.Tech. Civil with Str. Engg Semester: II Program: Civil Engineering

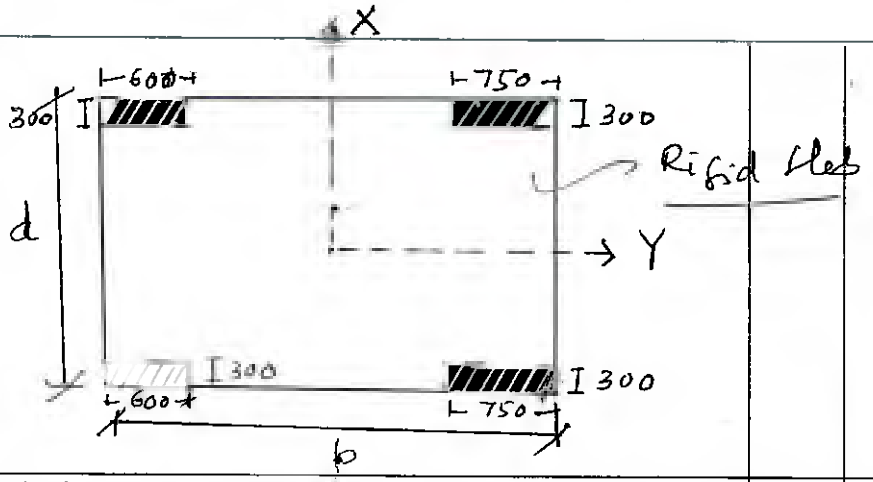
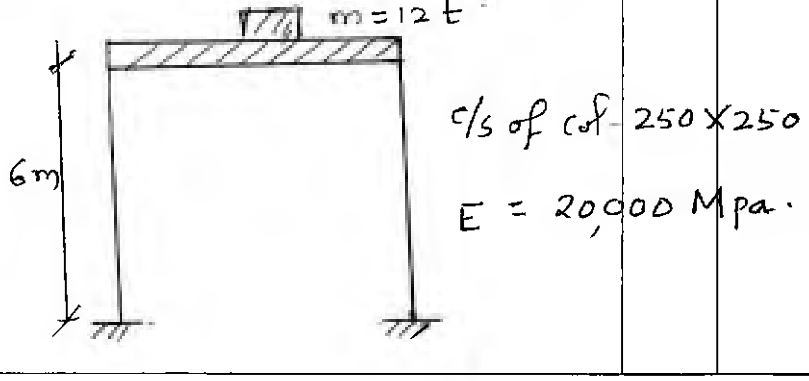
Name of the Course: Earthquake Engineering Course Code : PC-MSTE 202

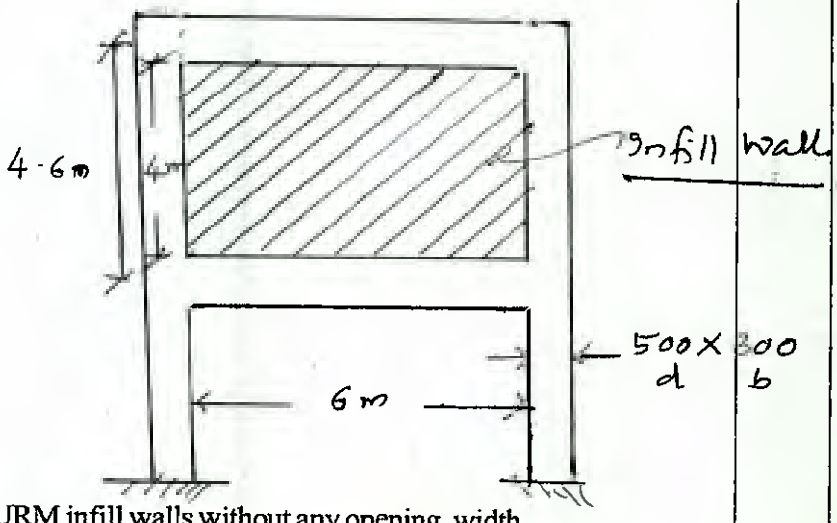
26/6/25

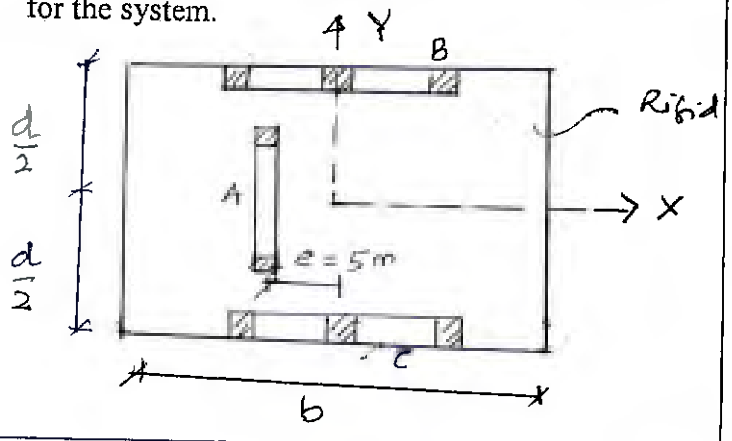
Instructions:

- Attempt any FIVE questions out of SEVEN questions.
- Answers to all sub questions should be grouped together.
- Figures to the right indicate full marks.
- Assume suitable data if necessary and state the same clearly

Question No		Max. Marks	Course outcome	Module No.
Q1 (a)	Answer the followings: (i) What is seismic zone factor? Explain briefly its significance	2	1	2
	(ii) Briefly explain the Plate Tectonic Theory of an earthquake occurrence	4	1	2
Q1 (b)	The plan of one story building is as shown in figure. The structure consists of a roof idealized as a rigid diaphragm, supported on four corner columns as shown in figure. The roof weight is uniformly distributed and has magnitude 200 kg/m^2. The plan dimensions are b= 30 m d=20m. Height of the building is 6m. (i) Derive the stiffness matrix and determine the natural frequencies and modes shapes of vibrations of the structure	8	2	1
	(ii) If the system is subjected to the ground motion only in X direction, the response spectrum of which is shown in figure1. Determine the design value of lateral deformation, base shear and bending moment for the system.	6	2,3	4

				
Q2 (a)	(i) A single-storey frame with a rigid girder as shown in the figure below is to be designed for ground motion, the response spectrum of which is shown in Figure 1. Determine the design value of lateral deformation and bending moments in the columns.	3	2,3	1,4
	(ii) If the columns of the frame are hinged at the base, determine the design values of lateral deformation and bending moments in columns. Comment on the influence of base fixity on the design deformation and bending moments.	3	2,3	1,4
	(iii)If the beam cross-section is much smaller than that of columns, so the beam stiffness can be neglected, and columns are fixed at the bottom, determine the design values of lateral deformation and bending moments in columns. Compare the design values with the case i &ii above.	4	2,3	1,4
				

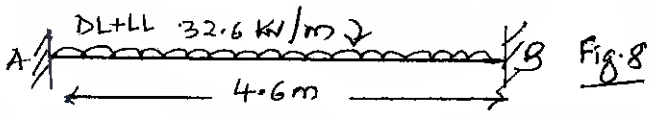
Q2 (b)	A two storey building frame with brick infill wall is shown in figure. Assuming the beams are rigid, calculate the lateral stiffness of the 2nd storey considering the effect of infill wall considering the infill was stiffness also as per IS 1893-2016. Compare the stiffness of the 2nd storey of frame with and without infill wall.  For URM infill walls without any opening, width w_{ds} of equivalent diagonal strut (*see* Fig. 7) shall be taken as: $w_{ds} = 0.175\alpha_h^{-0.4} L_{ds}$ where $\alpha_h = h\left(\sqrt[4]{\frac{E_m t \sin 2\theta}{4E_f I_c h}}\right)$ $E_f = 5000\sqrt{f_{ck}}$. $f_{ck} = 25$ Mpa $E_m = 13800$ Mpa $t = 230$ mm. $h = 4$m	10	2,4	1.6
Q3 (a)	What is response spectrum? Briefly explain the characteristics of response spectrum.	4	3	3
Q3 (b)	The plan of one storey building is as shown in figure. The structure consists of a roof idealized as a rigid diaphragm, supported on three frames A, B, and C as shown. The roof weight is uniformly distributed and has magnitude 200 Kg/m^2. The lateral stiffness are K_y = 25000 KN/m for frame A and K_x =20000 KN/m for frames B and K_x = 30,000 for frame C The plan dimensions are b= 30 m d=20m. The height of building is 6 m. (i) In general how many degrees of freedom for this system? Identify those dof. (ii) Calculate the stiffness matrix and write down the	1 10	3 1,3	1 1,4

	equation of motion if the system is subjected to ground motion $u_{gx}(t)$ in X direction only. (iii) If Kx = 20,000KN/m for both frames B & C, and e= 0 and the system is subjected to the ground motion only in X direction, the response spectrum of which is shown in figure1. Determine the design value of lateral deformation, base shear and bending moment for the system.  	5	3	1,4
Q4 (a)	For a residential RCC special moment resisting building frame the seismic weights on floors are W_1 =300 KN, W_2= 300 KN and W_3= 250 KN. The ground story height is 4.0m and first and second story height is 3.2m. The building is founded on hard soil and situated in zone IV. Determine the distribution of lateral forces and story shear by using equivalent static method. Use the response spectra given in figure1	8	4	5
Q4(b)	A three-storey frame has the following free vibration characteristics. The frame is to be designed for the ground motion characterized by the design spectrum given in figure 1 but scaled to a peak ground acceleration of 0.25g. Calculate the design values of lateral deformation of floors. Calculate also the storey drifts.	12	4	5

Story No.	Mass No.	Mass (t)	ω rad/sec	Mode shapes		
				Φ_{i1}	Φ_{i2}	Φ_{i3}
1	1	160	7.12	1.0	1.260	4.0
2	2	80	15.55	1.0	0.0	-1.0
3	3	40	20.81	1.0	-1.264	4.0

Q5 (a)	Explain the three requirements of displacement design of structure for earthquake load as per IS 1893-2016.	4	4	5

<table>
<tr><td>Q5 (b)</td><td>As per IS 1893-2016, how many mode need to be considered in the earthquake force calculation by Response Spectrum Method</td><td>2</td><td>4</td><td>5</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Q5 (c)</td><td>State the limitations of Equivalent static Method. As per IS 1893-2016, under what conditions the Equivalent static Method is permitted to use to calculate the earthquake forces.</td><td>2</td><td>4</td><td>5</td></tr>
<tr><td>Q5 (d)</td><td>Using response spectrum method, calculate the seismic force on each floor of the frame whose pre vibration properties are given below. Use the following additional data:
Z=0.24, I =1.2, R=4.0 and ξ = 5%. Assume foundation strata as soft and use response spectrum given in figure 3
Assume the story height as 4m for all story
<table>
<tr><td>Story No.</td><td>Mass No.</td><td>Mass (t)</td><td>ω rad/sec</td><td colspan="3">Mode shapes</td></tr>
<tr><td></td><td></td><td></td><td></td><td>Φ_{i1}</td><td>Φ_{i2}</td><td>Φ_{i3}</td></tr>
<tr><td>1</td><td>1</td><td>50</td><td>14.52</td><td>1.00</td><td>2.160</td><td>3.313</td></tr>
<tr><td>2</td><td>2</td><td>50</td><td>31.05</td><td>1.00</td><td>0.893</td><td>-1.473</td></tr>
<tr><td>3</td><td>3</td><td>40</td><td>46.10</td><td>1.00</td><td>-1.042</td><td>0.410</td></tr>
</table></td><td>12</td><td>4</td><td>5</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Q 6(a)</td><td>What is ductility? Explain the importance of ductility in earthquake resistant structures.</td><td>3</td><td>4</td><td>6</td></tr>
<tr><td>Q 6(b)</td><td>Briefly explain the following:
(i) When the structure is subjected to torsion ? Briefly explain the IS 1893-2016 provisions for design for torsion.
(ii) Explain briefly the plan irregularities of building as per IS 1893-2016.</td><td>6</td><td>4</td><td>5</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Q 6(c)</td><td>(i) A building having a non-uniform distribution of mass is shown in figure 4. Locate its center of mass
(ii) The plan of a simple one-storied building shown in figure 5. The stiffness of each column is indicated in figure. Obtain the center of stiffness. (Centre of Rigidity).</td><td>4</td><td>4</td><td>5</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Q 6(d)</td><td>The first floor plan of a building with shear walls is as shown in figure 6. The plinth level plan is also same. Calculate the fundamental period of the building as per the provision of 7.6.2, of IS 1893-2016 both in X and Y direction. The total height of building is 24 m.</td><td>4</td><td>4</td><td>5</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
</table>

Q 6(e)	For the SMRFs idealized as shear building with rigid girders shown in figure 7, investigate whether the building structure has soft story. The height of first story is 4.5 m and that of remaining is 3.0 m.	2	4	5
Q 7(a)	Explain the provisions of IS 13920-2016, for (i)Columns: longitudinal reinforcement, and transverse reinforcement only (splicing requirements need not be answered)	5	4	6
Q 7(b)	A beam of a frame is shown in figure 8 with limiting moment of resistance at both ends and for both sway from left right and right to left. Calculate the design shear force as per IS 13920 -2016. The calculated factored shear force is 142 KN  Fig 8 for sway from left to right $M^{AS}_{u\,lim}$ = 139 KN.m $M^{BH}_{u\,lim}$ = 139 KN.m ; For sway from right to left $M^{AH}_{u\,lim}$ = 240 KNm $M^{BS}_{u\,lim}$ = 65 KNm	5	4	6
Q 7(c)	For a column of cross section 530mm x 300mm calculate the special confinement reinforcement (i.e diameter and spacing of ties as a special confinement reinforcement) as per IS 13920-2026. The grade of concrete is M 25 and that of steel is Fe 415, consider clear cover to the ties is 60 mm. Also calculate the length l_0 of special confinement reinforcement. The clear height of column is 4.0 m.	5	4	6
Q 7(d)	Explain in brief the different types of bracings with neat sketches that can be used as lateral load resisting systems in steel structures	5	4	7

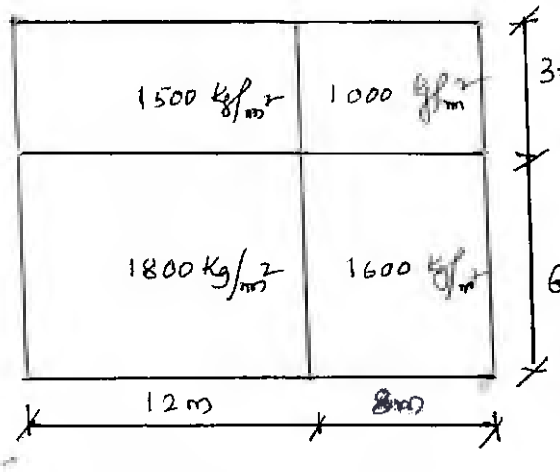


Fig 4

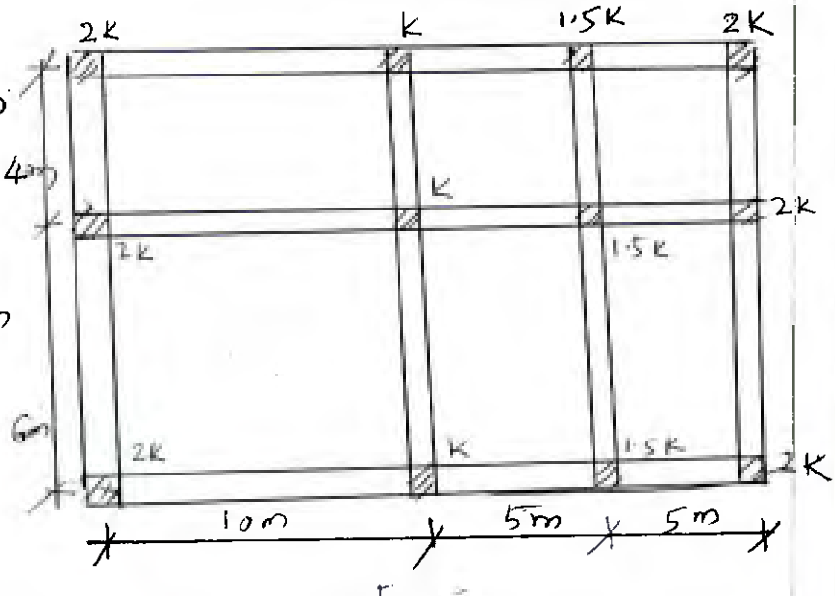
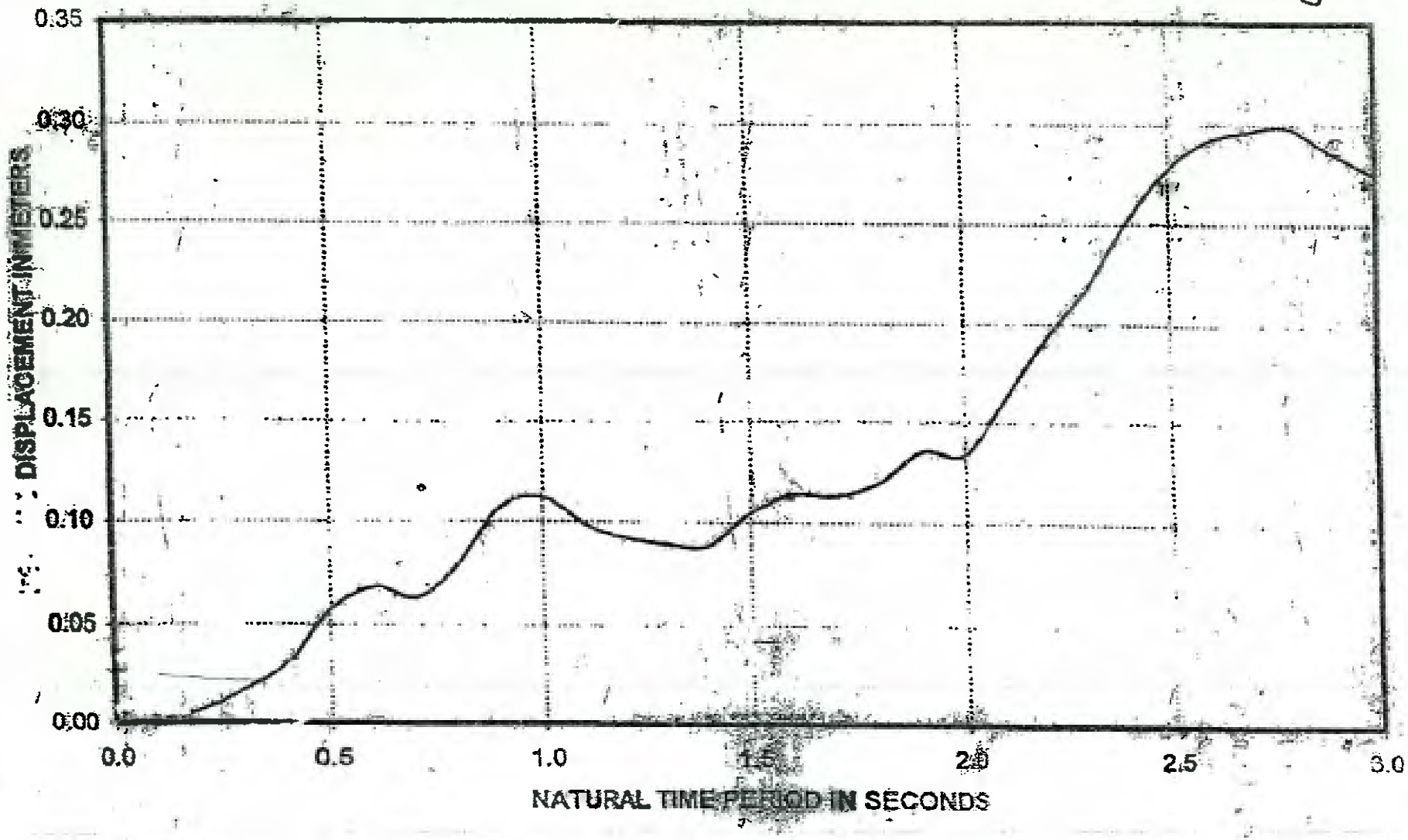


Fig 5

DISPLACEMENT RESPONSE SPECTRA
FOR EL-CENTRO EARTHQUAKE FOR 5% DAMPING PGA = 0.32g



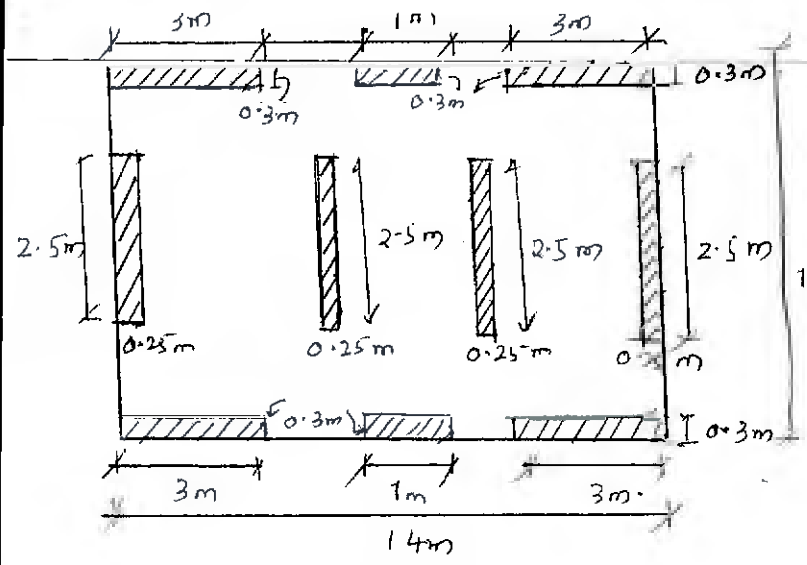


Fig. 6

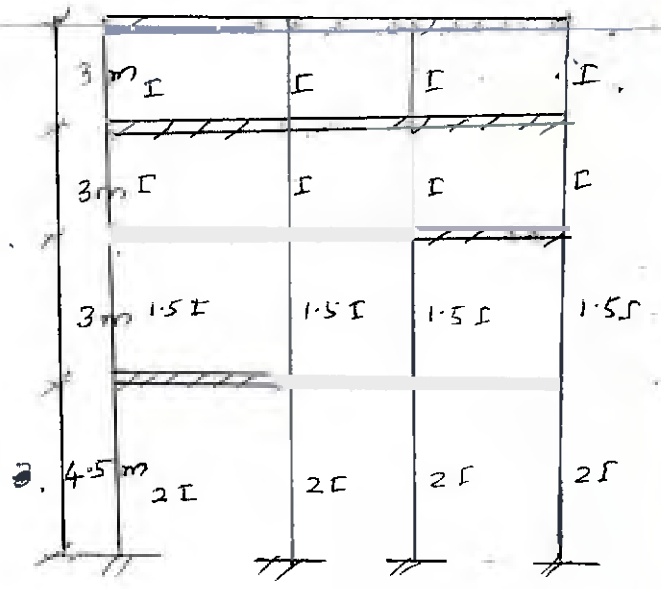


Fig. 7

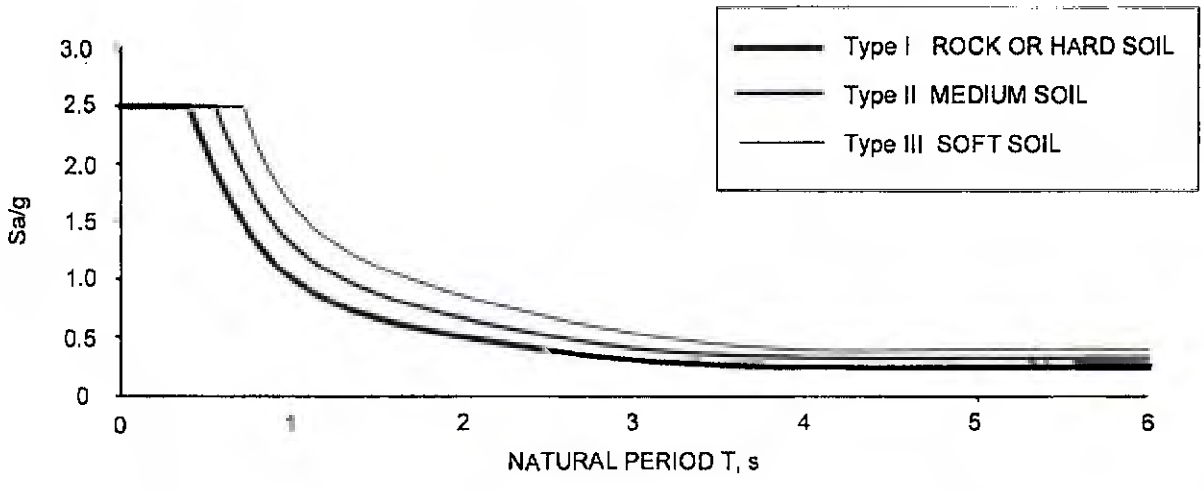


Fig. 2 2A SPECTRA FOR EQUIVALENT STATIC METHOD

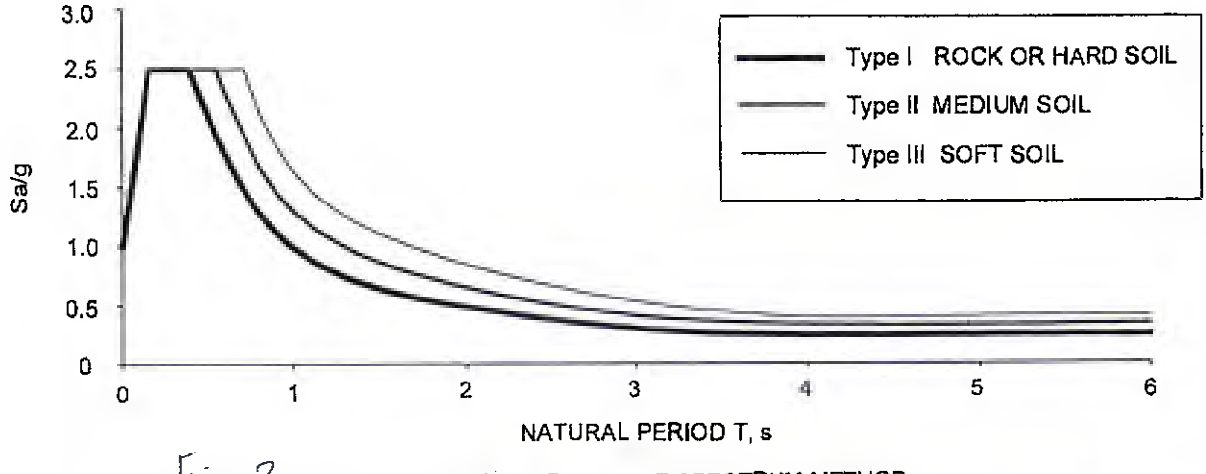


Fig 3 2B SPECTRA FOR RESPONSE SPECTRUM METHOD

SARDAR PATEL COLLEGE OF ENGINEERING

(Government Aided Autonomous Institute)
Munshi Nagar, Andheri (W) Mumbai – 400058

End Semester/Re-examination –May /June 2025 Examinations

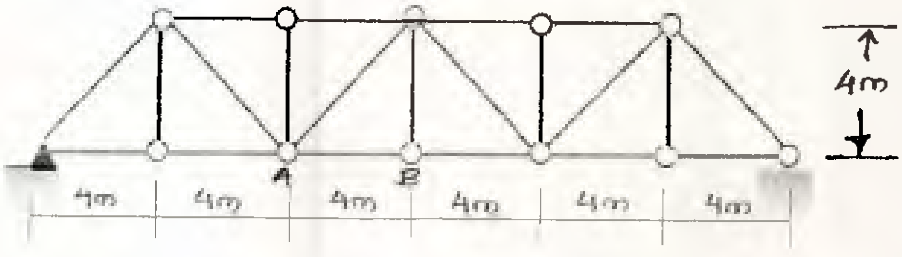
Program: M.Tech Civil Engg. (Structure) **Duration: 3Hours**

Course Code: PE-MTSE211 **Maximum Points: 100**

Course Name: Bridge Engineering **Semester: II**

Notes: 1) Attempt any 5 main questions
2) Assume suitable data if missing and mention the same clearly
3) Draw neat sketches wherever possible
4)Use of IRC 6,IRC 112 is permissible

Q.No.	Questions	Points	CO	BL	Module
1.a)	Enlist the different methods for bridge construction. Explain the balanced cantilever construction of bridges with neat sketches. Explain the suitability of this method for constructing bridges.	10	2,4	3	6
1.b)	Explain the data that needs to be collected for planning and design of a bridge.	10	1	2	1
2.a)	Design an RC slab culvert for an effective span of 5.5m and a carriageway of 9m. Wearing coat is of 80mm thickness. Consider single train of IRC Class 70R tracked vehicle loading. SIDL 10kN/m² Use M30 and Fe500	20	1,3,5	4	2,3
3	A steel struss bridge is to be designed for the following details The total dead load of truss per each nodal point = 150kN Deck width = 8.5m The bridge is to be used as foot over bridge. Design the member AB (assume a built-up section) considering live loads as 5kN/m² and slab thickness as 150mm of RCC. Use fy=250N/mm²	20	3	4	2,4



4.a)	Explain the various load cases and conditions to be considered for design of piers and foundations	06	4	4	5
4b)	For bridge pier having seismic mass at top as 5000kN, the height of pier is 10m and the cross section is 2.5m diameter made of M40 concrete. Assume the base to be fixed and obtain : i) Time period as per IRC ii) Base shear acting on the pier	10	1,5	3	2,7
4.c)	Explain the components and suitability of the following with neat sketches : i) Cable stayed bridges ii) Suspension bridges	04	3	2	6
5.a)	Explain in detail –Functions of bearings and its types	10	1	2	5

5.b)	Evaluate the axial load and moment carrying capacity for a pier having following data : Size : 1250mm x 1250mm Reinforcement on each face = 6nos-32mm dia(each face) Effective cover = 60mm Assume N.A = 550mm from extreme compression fibre Concrete grade is M45 and steel grade is Fe500.	10	3	4	5
6.a)	Obtain the maximum bending moment and maximum shear force for a girder of span 18.5m subjected to a single class 70R wheeled vehicle and having a courbon factor of 0.55	7	1,5	4	2,3
6.b)	Design an RCC girder of span 18.5m as per IRC 112 with following specifications: UDL on girder due to footpath and crash barrier = 8kN/m UDL on girder due to wearing course = 2.5kN/m Live load as per Q.6(a) Effective slab width as beam flange = 2.1m Depth of slab = 0.21m Use M35 and Fe500	13	3	4	3
-	Design a shallow foundation as per IRC 112 for a pier of size 1.25m x 1.25m. The design axial load = 3100kN and design moment along transverse axis = 1900kNm. SBC of soil = 160kN/m^2. Use M30 and Fe500. Provide checks for : a) Flexure b) One way shear c) Punching shear at distance 2 x depth of footing from face of pier and at face of pier	20	3,5	4	5

SARDAR PATEL COLLEGE OF ENGINEERING

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~~End-Semester~~ /Re-examination –~~May~~ /June 2025 Examinations

Program: M.Tech Civil Engg. (Structure) Sem II

Course Code: PE-MTSE211

Course Name: Bridge Engineering

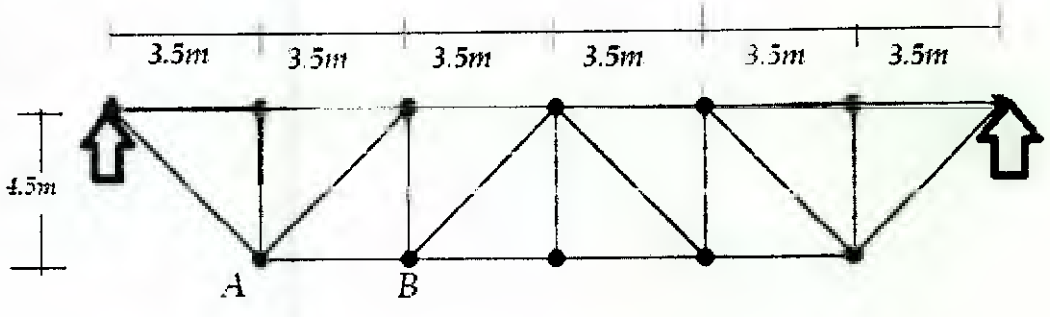
Duration: 3Hours

Maximum Points: 100

Semester: II

27/6/25

Notes: 1) Attempt any 5 main questions
2) Assume suitable data if missing and mention the same clearly
3) Draw neat sketches wherever possible
4)Use of IRC 6,IRC 112 is permissible

Q.No.	Questions	Points	CO	BL	Module
1.a)	Enlist the various methods of bridge construction. Explain segmental construction method and the importance of construction stage analysis in the design of such bridges.	10	2,4	3	6
1.b)	Explain in detail the key points that affect bridge aesthetics	10	1	2	1
2.a)	Design an RC slab culvert for a clear span of 6m and a carriageway of 9m. Wearing coat is of 60mm thickness. Consider single train of IRC Class 70R wheeled vehicle loading. SIDL = 5kN/m^2. Use M30 and Fe500	20	1,3,5	4	2,3
3	**For the steel truss shown below :** The total dead load per nodal point = 200kN Deck width = 7.5m Design the member AB (assume a built-up section) as marked when a single IRC class 70R tracked vehicle is passing on the bridge	20	1,3,5	4	2,4
					
4.a)	Calculate the forces in each pile using rivet theory if the axial compression load is 3000kN and Moment about transverse axis is 2500kNm. There are 6 piles arranged in 3*2 manner (3 piles in each row are parallel to the transverse direction) and c/c distance between all piles is 2.2m in both directions.	06	4	4	5
4b)	For bridge pier having seismic mass at top as 4000kN, the height of pier is 8m and the cross section is 1.5m diameter made of M40 concrete. Assume the base to be fixed and obtain : i) Time period as per IRC ii) Base shear acting on the pier	10	1,5	2	2,7

4.c)	Explain the provision to account for dynamic effect of imposed load in bridges. Explain the factors influencing it.	04	3	2	1
5.a)	Explain in detail – Bearing articulation for straight and curved bridges	10	1	2	5
5.b)	Evaluate the axial load and moment carrying capacity for a pier having following data : Size : 1200mm x 1200mm Reinforcement on each face = 6nos-25mm dia(each face) Effective cover = 60mm Assume N.A = 500mm from extreme compression fibre Concrete grade is M45 and steel grade is Fe500.	10	3	4	5
6.a)	Obtain the maximum bending moment and maximum shear force for a girder of span 18m subjected to a single class 70R wheeled vehicle and having a courbon factor of 0.35	7	1,5	4	2,3
6.b)	Design an RCC girder of span 18m as per IRC 112 with following specifications: UDL on girder due to footpath and crash barrier = 10kN/m UDL on girder due to wearing course = 3.5kN/m Live load as per Q.6(a) Effective slab width as beam flange = 2.0m Depth of slab = 0.2m Use M35 and Fe500	13	3	4	3
7	Design a shallow foundation as per IRC 112 for a pier of size 1.2m x 1.2m. The design axial load = 3000kN and design moment along transverse axis = 1800kNm. SBC of soil = 150kN/m^2. Use M35 and Fe500. Provide checks for : a) Flexure b) One way shear c) Punching shear at distance 2 x depth of footing from face of pier and at face of pier	20	3,5	4	5

Bharatiya Vidya Bhavan's

SARDAR PATEL COLLEGE OF ENGINEERING

(Government Aided Autonomous Institute)
Munshi Nagar, Andheri (W) Mumbai – 400058

TERM END EXAMINATION ~~JULY~~ MAY 2025

Program: M. tech Civil (Structural engineering) Sem II **Duration: 3 Hr**

Course Code: PE-MTSE~~131~~ 221 **Maximum Points: 100**

Course Name: Advanced design of concrete Structures **Semester: II**

Notes: 1) Each question carries 20 marks

2) Solve any five questions out of seven questions

28/5/25

Q.No.	Questions	Points	CO	BL	Module
Q. 1(a)	For a doubly reinforced beam of size 23 cm x 45 cm (overall depth) is provided with 3-20 mm dia steel as compression and tension reinforcement at an effective cover of 5 cm each, calculate ultimate moment carrying capacity of beam. The concrete has a 28-day cube strength of 250 kg/cm^2 and steel has compressive and tensile yield stress of 2500 kg/cm^2 and 2800 kg/cm^2 respectively. **Use ultimate load method**	14	CO1	L5	1
Q.1(b)	What are different limit states used for design ? Explain characteristic strength and characteristic load for limit state design.	06	CO1	L2	2
Q. 2	For the 3 equal span continuous beam of each span 6.0 m is subjected to service UDL of 3.0 T/m including self weight. Use load factor 2.0. Size of beam : 23 cm x 60 cm effective depth σcu= 200 kg.cm2 σsy =2400 kg/cm2 ε sy = 0.0012 Tensile steel 4- 20 dia bars **Use Baker's method of design** a) Calculate EI b) Calculate plastic moment c) Calculate influence coefficients d) Calculate relative rotation and permissible rotation e) Check for increase in capacity of section	20	CO1	L3	2

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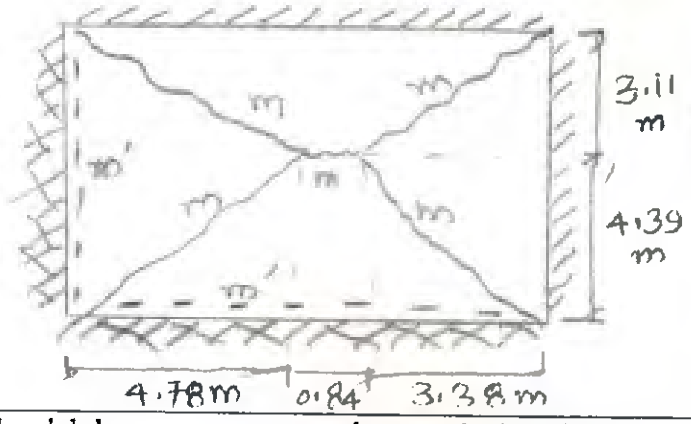
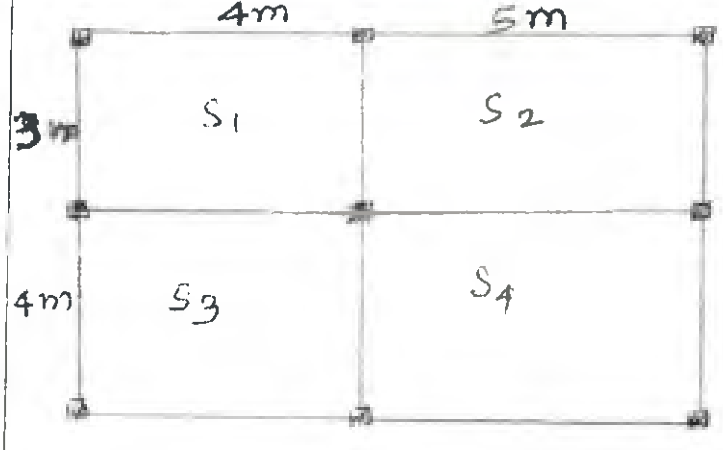
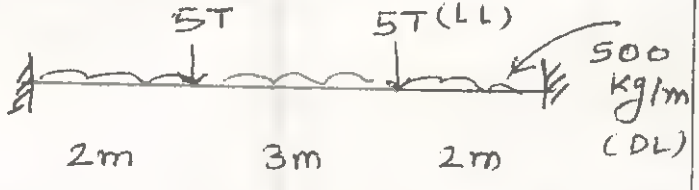
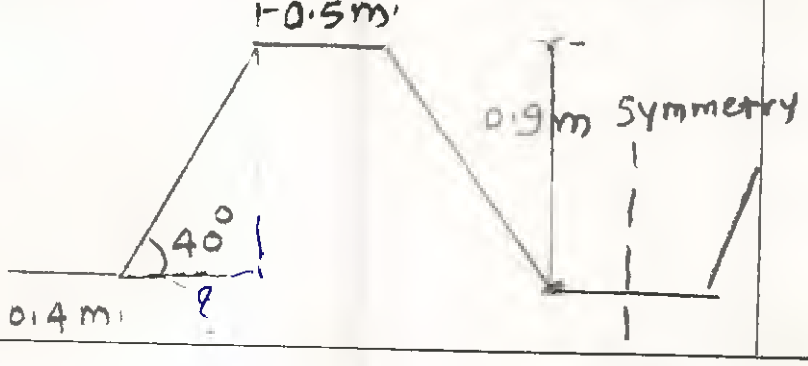
TERM END EXAMINATION ~~JULY~~ 2025

Q.3(a)	What is plastic hinge? Explain the concept of tensile hinge and compression hinge used in Baker's method of analysis	06	CO1	L2	.2
Q.3(b)	Using the Virtual Work Method, analyze a 250 mm thick reinforced concrete slab as shown in figure below with pattern of yield line as shown. The slab is subjected to the ultimate uniform load of intensity 20 kn/m2. Assume isotropic reinforcement with equal 'm' in each direction and m = m'. Calculate value of m 	14	CO1	L5	3
Q.4(a)	For the slab beam arrangement shown calculate design bending moments for slab after redistribution of moments.The slabs are subjected to live load of 3.0 Kn/m2 in addition to floor load 1 Kn/m2 and self-weight. **Design of slab is not required.** 	06	CO1	L5	4

Q.4(b)	For the beam shown with the service load, Use load factors 1.8 for dead load and 2.2 for live load. The negative plastic moment at support is equal to positive plastic moment at midspan. Design the beam using **Cambridge method** approach. Give checks for rotation required and rotation capacity available. The concrete has a 28-day cube strength of 200 kg/cm^2 and steel has tensile yield stress of 2500 kg/cm^2 and 2800 kg/cm^2 Give check for serviceability as well. 	14	CO1	L3	2
Q. 5	Analyze intermediate panel of flat slab and Calculate design bending moments for slab of size 6.0 m x 7.0 m. The slab is subjected to live load of 4.0 Kn/m2 in addition to floor load 1 Kn/m2 and self-weight The slab is supported by columns of size 500 mm x 500 mm . Provide drop panel. Use M30 grade of concrete and mild steel reinforcement Fe 415. **Use direct design method** **Draw reinforcement in plan**	20	CO2	L6	5
Q.6	Perform **preliminary analysis up to stress calculations at joints** for the folded plate shown. Thickness of plate is 110 mm. Loading on inclined plate 200 kg/m2 and loading on horizontal plate 250 kg/m2. Length of plate is 20 m. 	20	CO2	L5	7

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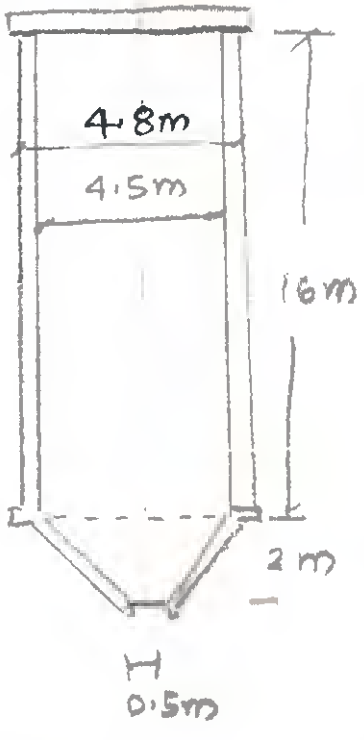
TERM END EXAMINATION MAY JULY 2025

Q. 7	Design Silo with conical hopper bottom to store wheat. The dimensions of silo are as shown in the figure. Design ring beam also. **Use Airy's theory** Assume unit weight of wheat as 8340 N/m3. Use M20 grade of concrete and mild steel reinforcement Fe 250. Perform calculations at h = 4 m, 8m, 12m, and 16m Use $\mu = 0.46$, $\mu' = 0.43$ **Draw reinforcement in section** 	20	CO2	L6	6

Horizontal Pr in shallow Portion

$$ph = wh\left[\frac{1}{\sqrt{1+u'^2+\sqrt{u}(u+u')}}\right]^2$$

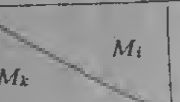
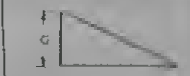
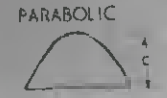
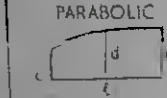
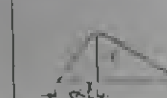
Horizontal pr. in deep Portion

$$ph = \frac{wb}{(u+u')}\left[1-\frac{\sqrt{1+u'^2}}{\sqrt{\frac{2h}{b}(u+u')+(1-uu')}}\right]$$

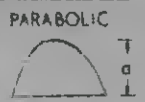
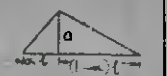
Horizontal force

$$ph = \frac{wb^2}{2(u+u')^2}\left[\sqrt{\frac{2h}{b}(u+u')+1(1-uu')}-\sqrt{1+}\right.$$

TABLE 8.1 PRODUCT

M_k \ M_l			
	lac	$\frac{1}{2}\,lac$	$\frac{1}{3}\,lac$
	$\frac{1}{2}\,lac$	$\frac{1}{3}\,lac$	$\frac{1}{4}\,lac$
	$\frac{1}{2}\,lac$	$\frac{1}{6}\,lac$	$\frac{1}{3}\,lac$
PARABOLIC 	$\frac{2}{3}\,lac$	$\frac{1}{3}\,lac$	$\frac{1}{4}\,lac$
	$\frac{1}{2}\,lac$	$\frac{1}{4}\,lac$	$\frac{1}{4}\,lac$
	$\frac{1}{2}\,la\,(c+d)$	$\frac{1}{2}\,la\,(2c+d)$	$\frac{1}{6}\,la\,(c+2d)$
PARABOLIC  d = central ordinate	$\frac{1}{6}\,la\,(c+4d+e)$	$\frac{1}{6}\,la\,(c+2d)$	$\frac{1}{6}\,la\,(2d+e)$
	$\frac{1}{2}\,lac$	$\frac{1}{6}\,lac\,(2-\alpha)$	$\frac{1}{6}\,lac\,(1+\alpha)$

INTEGRALS--$\int M_l M_k ds$ Values

PARABOLIC 			
$\frac{2}{3}\,lac$	$\frac{1}{2}\,lac$	$\frac{1}{2}\,l\,(a+b)\,c$	$\frac{1}{3}\,lac$
$\frac{1}{3}\,lac$	$\frac{1}{4}\,lac$	$\frac{1}{6}\,l\,(2a+b)\,c$	$\frac{1}{12}\,lac\,(2-\alpha)$
$\frac{1}{6}\,lac$	$\frac{1}{4}\,lac$	$\frac{1}{6}\,l\,(a+2b)\,c$	$\frac{1}{12}\,lac\,(1+\alpha)$
$\frac{5}{12}\,lac$	$\frac{1}{12}\,lac$	$\frac{1}{3}\,l\,(a+b)\,c$	$\frac{1}{12}\,lac\,(\alpha+1-\alpha^2)$
$\frac{7}{12}\,lac$	$\frac{1}{4}\,lac$	$\frac{1}{3}\,l\,(a+b)\,c$	*If* $\alpha = 0{\cdot}5$ $\frac{1}{12}\,lac\,\frac{(3-4\alpha^2)}{(1-\alpha)}$ *If* $\alpha > 0{\cdot}5$ $\frac{lac}{12\alpha}\,(8\alpha - 4\alpha^2 - 1)$
$\frac{1}{12}\,la(c+d)$	$\frac{1}{12}\,la\,(c+d)$	$\frac{1}{6}\,l\,[a\,(2c+d)$ $+\,b\,(2d+c)\,]$	$\frac{1}{12}\,al\,[d\,(1-\alpha)+$ $c\,(2-\alpha)\,]$
$\frac{la}{15}\,(c+c+8d)$	$\frac{la}{24}\,(c+10d+e)$	$\frac{1}{6}[\frac{1}{2}la\,(2d+c)$ $+\frac{1}{2}lb\,(2d+e)\,]$	$\frac{1}{12}\,al\,\times$ $[2\alpha^2(c-2d+e)$ $+\,4\alpha\,(d-1)$ $+\,2c+4d\,]$
$\frac{1}{3}\,lac\,(1+\alpha-\alpha^2)$	$\alpha = 0{\cdot}5$ $\frac{1}{12}lac\,\frac{(3-4\alpha^2)}{1-\alpha}$ $\alpha > 0{\cdot}5$ $\frac{lac}{12\alpha}\,(8\alpha - 4\alpha^2 - 1)$	$\frac{1}{6}lc\,[b\,(1+\alpha)$ $+\,a\,(2-\alpha)\,]$	$\frac{1}{3}\,lac$

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(Government Aided Autonomous Institute)

Munshi Nagar, Andheri (W) Mumbai – 400058

Re- Examination MAY June 2025

~~TERM END~~ EXAMINATION JULY 2025

Program: M. tech Civil (Structural engineering) Sem II

Course Code: PE-MTSE131

Course Name: Advanced design of concrete Structures

Duration: 3 Hr

Maximum Points: 100

Semester: II

Notes: 1) Each question carries 20 marks

2) Solve any five questions out of seven questions

30/6/25

Q.No.	Questions	Points	CO	BL	Module
Q. 1(a)	For a doubly reinforced beam of size 25 cm x 40 cm (overall depth) is provided with 4- 16 mm dia steel as compression and tension reinforcement at an effective cover of 5 cm each, calculate ultimate moment carrying capacity of beam. The concrete has a 28-day cube strength of 250 kg/cm^2 and steel has compressive and tensile yield stress of 2500 kg/cm^2 and 2800 kg/cm^2 respectively. **Use ultimate load method**	14	CO1	L5	1
Q.1(b)	What are different limit states used for design ? Explain characteristic strength and characteristic load for limit state design.	06	CO1	L2	2
Q. 2	For the 3 equal span continuous beam of each span 5.0 m is subjected to service UDL of 2.5 T/m including self weight. Use load factor 2.0. Size of beam : 23 cm x 40 cm effective depth σcu= 200 kg.cm2 σsy =2400 kg/cm2 ε sy = 0.0012 Tensile steel 4- 16 dia bars **Use Baker's method of design** a) Calculate EI b) Calculate plastic moment c) Calculate influence coefficients d) Calculate relative rotation and permissible rotation e) Check for increase in capacity of secti	20	CO1	L3	2

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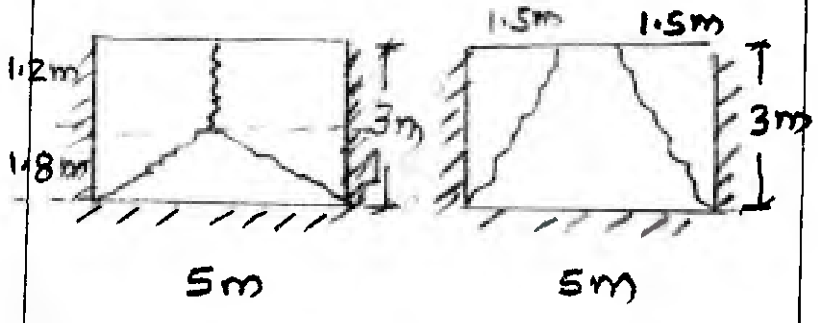
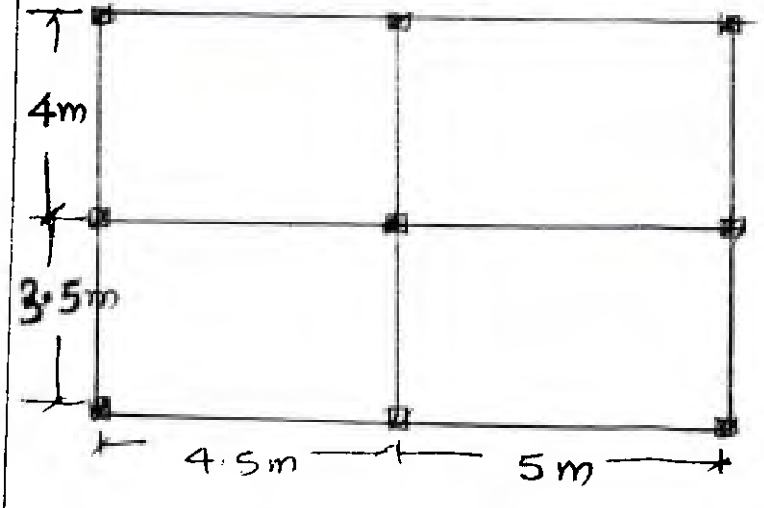
Q.3(a)	Explain the concept of tensile hinge and compression hinge used in Baker's method of analysis	06	CO1	L2	.2
Q.3(b)	Using the Virtual Work Method, analyze a 250 mm thick reinforced concrete slab as shown in figure below with pattern of yield line as shown. The slab is subjected to the ultimate uniform load of intensity 20 kn/m2. Assume isotropic reinforcement with equal 'm' in each direction and m = m'. Calculate value of m  	14	CO1	L5	3
Q.4(a)	For the slab beam arrangement shown calculate design bending moments for slab after redistribution of moments.The slabs are subjected to live load of 3.0 Kn/m2 in addition to floor load 1 Kn/m2 and self-weight. **Design of slab is not required.**  	06	CO1	L5	4

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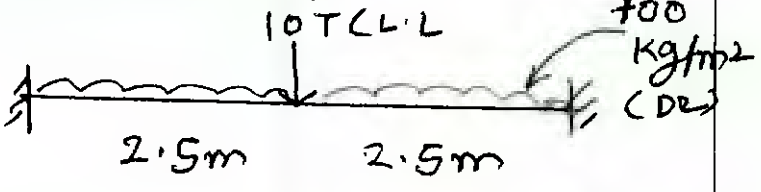
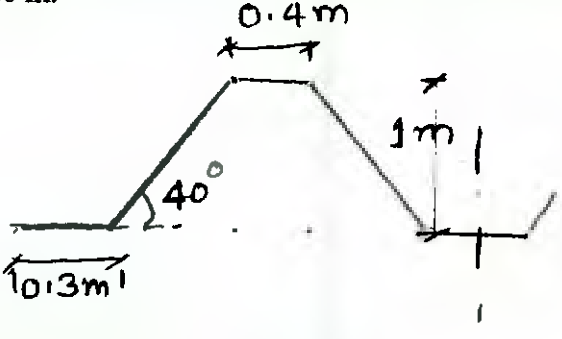
TERM END EXAMINATION JULY 2025

Q.4(b)	For the beam shown with the service load, Use load factors 1.8 for dead load and 2.2 for live load. The negative plastic moment at support is equal to positive plastic moment at midspan. Design the beam using **Cambridge method** approach. Give checks for rotation required and rotation capacity available. The concrete has a 28-day cube strength of 200 kg/cm^2 and steel has tensile yield stress of 2500 kg/cm^2 and 2800 kg/cm^2 Give check for serviceability as well. 	14	CO1	L3	3
Q. 5	Analyze intermediate panel of flat slab and Calculate design bending moments for slab of size 7.0 m x 9.0 m. The slab is subjected to live load of 4.0 Kn/m2 in addition to floor load 1 Kn/m2 and self-weight The slab is supported by columns of size 450 mm x 450 mm . Provide drop panel. Calculate reinforcement for M30 grade of concrete and Fe 415 steel **Use direct design method** **Draw reinforcement in plan**	20	CO2	L6	5
Q.6	Perform **preliminary analysis up to stress calculations at joints** for the folded plate shown. Thickness of plate is 110 mm. Loading on inclined plate 200 kg/m2 and loading on horizontal plate 250 kg/m2. Length of plate is 20 m. 	20	CO2	L5	7

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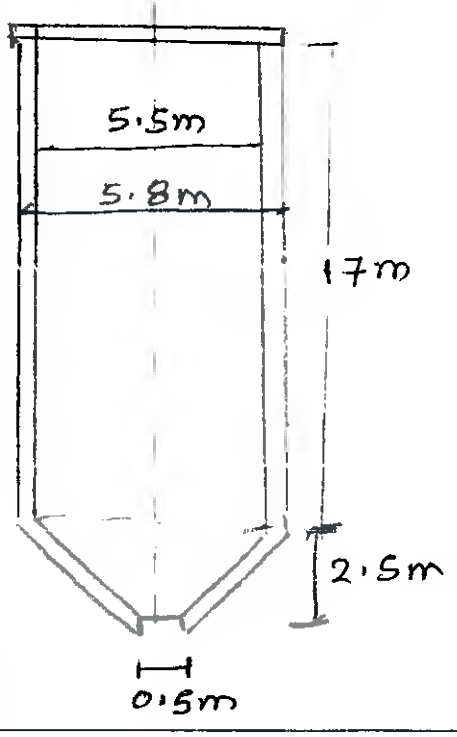
Re (RA) Time

~~TERM END~~ EXAMINATION JULY 2025

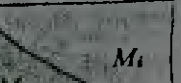
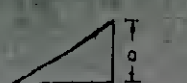
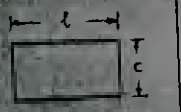
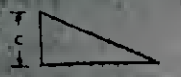
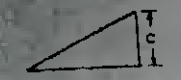
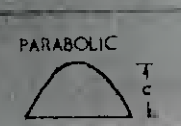
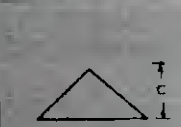
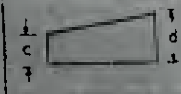
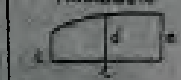
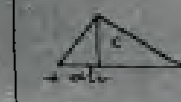
Q. 7	Design Silo with conical hopper bottom to store wheat. The dimensions of silo are as shown in the figure. **Use Janson's theory** Assume unit weight of wheat as 8340 N/m3. Use M20 grade of concrete and mild steel reinforcement Fe 250. Perform calculations at h = 4 m, 8m, 12m, and 17m Use $\mu = 0.46$ **Draw reinforcement in section** 	20	CO2	L6	6

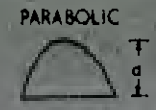
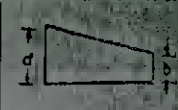
Horizontal Pr. -

$$P_h = \frac{wr}{\mu'_f}\left[1 - e^{-\frac{\mu'_f K_f h}{r}}\right]$$

$$P_v = \frac{P_h}{K_f} \qquad \phi = \tan^{-1}\mu \qquad \mu'_f = \tan(0.754$$

TABLE 8.1 PRODUCT INTEGRALS—$\int M_1 M_2 ds$ Values

M_1 \ M_2			
	lac	$\frac{1}{2}\,lac$	$\frac{1}{2}\,lac$
	$\frac{1}{2}\,lac$	$\frac{1}{3}\,lac$	$\frac{1}{3}\,lac$
	$\frac{1}{2}\,lac$	$\frac{1}{3}\,lac$	$\frac{1}{3}\,lac$
PARABOLIC 	$\frac{2}{3}\,lac$	$\frac{1}{3}\,lac$	$\frac{1}{4}\,lac$
	$\frac{1}{2}\,lac$	$\frac{1}{3}\,lac$	$\frac{1}{3}\,lac$
	$\frac{1}{2}\,la\,(c+d)$	$\frac{1}{3}\,la\,(2c+d)$	$\frac{1}{6}\,la\,(c+2d)$
PARABOLIC 	$\frac{1}{6}\,la\,(c+4d+e)$ d = central ordinate	$\frac{1}{6}\,la\,(c+2d)$	$\frac{1}{6}\,la\,(2d+e)$
	$\frac{1}{3}\,lac$	$\frac{1}{6}\,lac\,(2-\alpha)$	$\frac{1}{6}\,lac\,(1+\alpha)$

PARABOLIC 			
$\frac{2}{3}\,lac$	$\frac{1}{2}\,lac$	$\frac{1}{2}\,l\,(a+b)\,c$	$\frac{1}{2}\,lac$
$\frac{1}{2}\,lac$	$\frac{1}{3}\,lac$	$\frac{1}{6}\,l\,(2a+b)\,c$	$\frac{1}{6}\,lac\,(2-\alpha)$
$\frac{1}{3}\,lac$	$\frac{1}{6}\,lac$	$\frac{1}{6}\,l\,(a+2b)\,c$	$\frac{1}{6}\,lac\,(1+\alpha)$
$\frac{1}{4}\,lac$	$\frac{1}{5}\,lac$	$\frac{1}{10}\,l\,(a+b)\,c$	$\frac{1}{10}\,lac\,(\alpha+1-\alpha^2)$
$\frac{1}{5}\,lac$	$\frac{1}{6}\,lac$	$\frac{1}{12}\,l\,(a+b)\,c$	If $\alpha < 0.5$ $\frac{1}{12}\,lac\,\frac{(3-4\alpha^2)}{(1-\alpha)}$ If $\alpha > 0.5$ $\frac{lac}{12\alpha}(8\alpha-4\alpha^2-1)$
$\frac{1}{12}\,la(c+d)$	$\frac{1}{24}\,la\,(c+d)$	$\frac{1}{24}\,l\,[a\,(2c+d)+b\,(2d+c)]$	$\frac{1}{24}\,al\,[d\,(1+\alpha)+c\,(2-\alpha)]$
$\frac{la}{15}(c+4d+e)$	$\frac{la}{24}\,(c+10d+e)$	$\frac{1}{24}l[a\,(2d+c)+\frac{1}{2}b\,(2d+c)]$	$\frac{1}{24}\,al\times[2\alpha^2(c-2d+e)+4\alpha\,(d-c)+2c+4d]$
$\frac{1}{3}\,lac\,(1+\alpha-\alpha^2)$	$\alpha < 0.5$ $\frac{1}{12}\,lac\,\frac{(3-4\alpha^2)}{1-\alpha}$ $\alpha > 0.5$ $\frac{lac}{12\alpha}(8\alpha-4\alpha^2-1)$	$\frac{1}{12}\,lc\,[b\,(1+\alpha)+a\,(2-\alpha)]$	$\frac{1}{6}\,lac$